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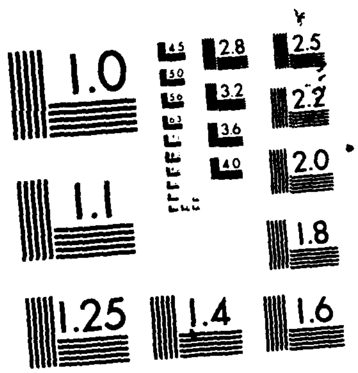
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POURING FLOWS

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POURING FLOWS

Jean-Marc Vanden-Broeck*,[1] and Joseph B. Keller**,[2]

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ABSTRACT

Free surface flows of a liquid poured from a container are calculated numerically for various configurations of the lip. The flow is assumed to be steady, two dimensional and irrotational, the liquid is treated as inviscid and incompressible, and gravity is taken into account. It is shown that there are flows which follow along the under side of the lip or spout, as in the well-known "teapot effect", which was treated previously without including gravity. Some of the results are applicable also to flows over weirs and spillways.

AMS (MOS) Subject Classification: 76B10

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[2]Research supported by the Office of Naval Research, the Air Force Office of Scientific Research, the Army Research Office, and the National Science Foundation.

SIGNIFICANCE AND EXPLANATION

The flow of a liquid poured from a container is a free boundary flow driven by gravity, and it has proved difficult to determine such flows analytically. Therefore we have developed a method to calculate them numerically.

Two calculated pouring flows from a thin-walled vessel are shown in Figures 1 and 2. The flow in Figure 1 is that desired when pouring a beverage like tea, while the flow in Figure 2 is the undesirable one which occurs in the "teapot effect".

Flows which run along a wall are desirable when pouring beer to avoid excess foam, and when pouring acid to avoid splashing. All these flows are like the calculated flow shown in Figure 3, which also represents the flow in a spillway.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

POURING FLOWS

Jean-Marc Vanden-Broeck*,1 and Joseph B. Keller**,2

1. Introduction

The flow of a liquid poured from a container is a free boundary flow driven by gravity, and it has proved difficult to determine such flows analytically. Therefore we have developed a method to calculate them numerically when the flow is steady, two dimensional and irrotational, and the fluid is inviscid and incompressible. We have used it before to calculate flows over weirs [1], and now we shall adapt it to the calculation of flows over lips or spouts of various shapes. Some of our results are applicable also to weirs and spillways.

Two calculated pouring flows from a thin-walled vessel are shown in Figures 1 and 2. In both cases the angle β between the vessel wall and the horizontal is $\beta = \pi/3$. The flow in Figure 1 is that desired when pouring a beverage like tea, while the flow in Figure 2 is the undesirable one which occurs in the "teapot effect". It was analyzed previously without taking account of gravity [2], and the present calculation shows that such a flow can occur also when gravity is taken into

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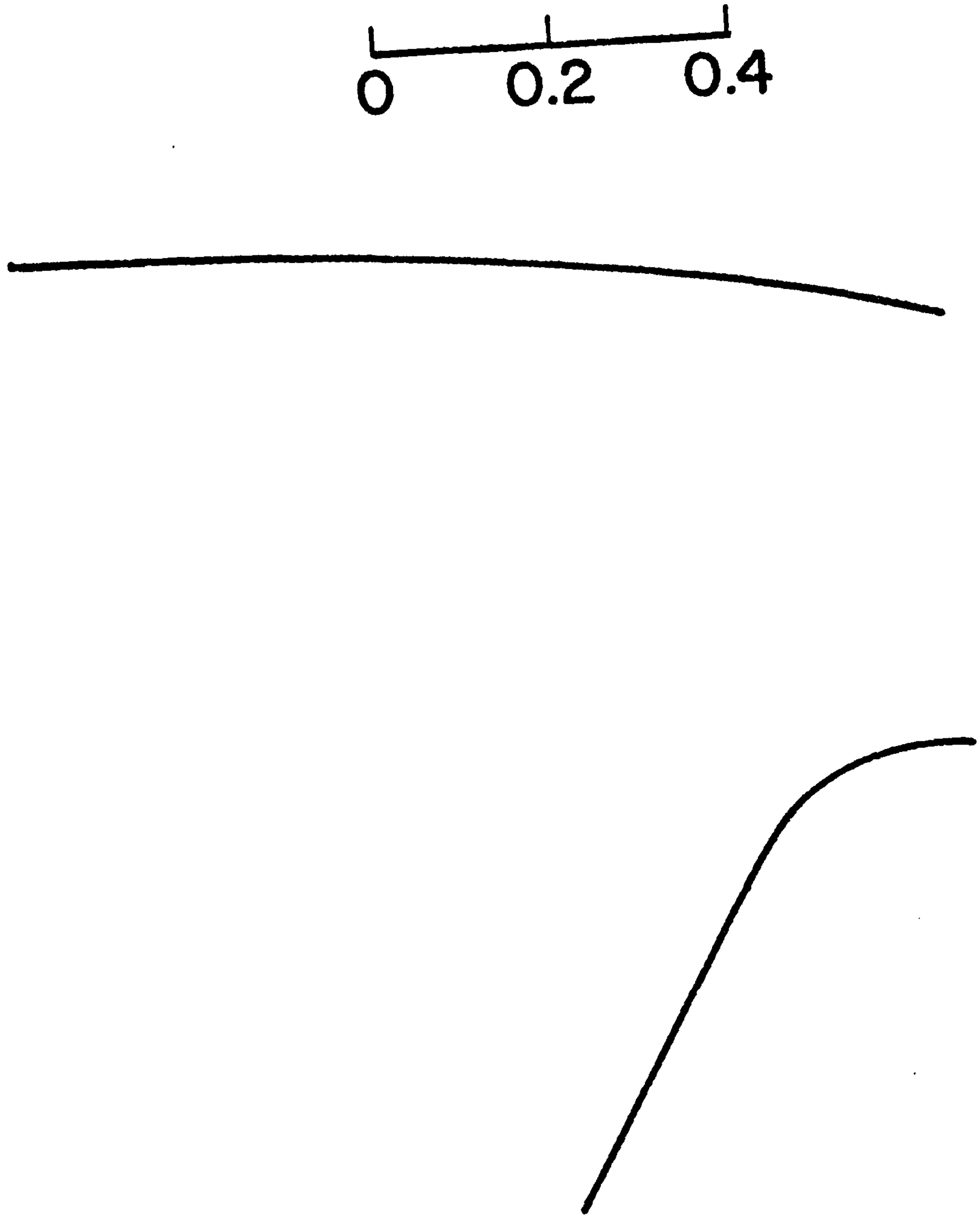

Figure 1. Liquid pouring over a thin wall, or flowing over a thin weir. The wall slopes at the angle β from the horizontal. The flow shown here, which has two free streamlines, was calculated for $\beta = \pi/3$ by the first method in Section 5.

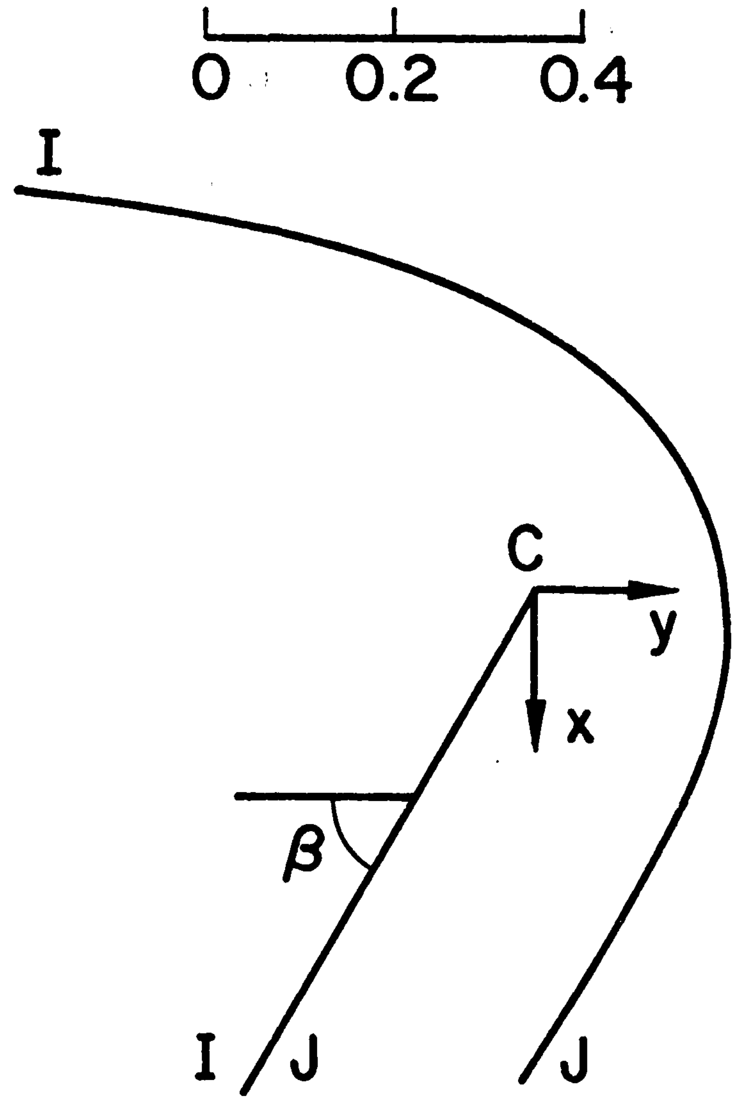

Figure 2. The undesirable "teapot effect" flow over the same thin wall or weir as in Figure 1. The flow shown has one free streamline. It was calculated for $\beta = \pi/3$ by the second method in Section 5. The x,y coordinates are shown with their origin at C, the end of the wall. The points I and J are at infinity on opposite sides of the wall.

account.

A flow like that in Figure 1, but with $\beta = \pi/2$, was calculated in [1] to describe the flow over a thin vertical weir in a deep channel. In the same way, the flow in Figure 1 applies to a thin weir sloping at the angle $\beta = \pi/3$ in a deep channel. Flows for other slopes can be calculated by the present method.

Flows which run along a wall are desirable when pouring beer to avoid excess foam, and when pouring acid to avoid splashing. But then the flow must run along the inner wall of the receiving container, rather than along the outer wall of the pouring container. In chemical laboratories to avoid splashing, liquids are often poured onto stirring rods which they run along. All these flows are like the calculated flow shown in Figure 3, which also represents the flow in a spillway. For it the angle between the two walls is $\beta = 3\pi/4$ and the Froude number $F = U(gH)^{-1/2}$ is $F = 1.3$.

Similar flows can be calculated for any $F > 1$ and for any value of β satisfying $0 \leq \beta \leq \pi$. Another example, shown in Figure 4 for $\beta = \pi/4$ and $F = 1.3$, represents the undesirable flow over a lip which has its sides meeting at the angle β. Goh and Tuck [3] have calculated the desirable flow from a spout consisting of two horizontal plates.

In Section 2 we shall formulate the problem for the flow shown in Figure 4 for any $F > 1$ and $\beta \leq \pi$, and we shall present the analytical solution of this problem for $F = \infty$ and show how to obtain the asymptotic form of the solution for $F \gg 1$. In Section 4 we describe our numerical results, which yield the free streamlines shown in Figures 3 and 4. In Section 5 we indicate how to modify the method presented in Section 2 in order to treat the flows shown in Figures 1 and 2.

2. Formulation

Let us consider the flow in the region of the z-plane shown in Figure 4. The horizontal bottom IC is a streamline on which we require that the stream function $\psi = 0$. The wall CJ, which slopes at an angle β from the horizontal, is part of the same streamline. We choose cartesian coordinates with the y-axis along the horizontal wall IC and the x-axis directed vertically downwards. The origin is at the corner C and gravity acts in the positive x direction. As $x \to +\infty$, the flow approaches the thin jet flow of Keller and Weitz [4] and Keller and Geer [5]. As $y \to -\infty$, the flow approaches a uniform stream with a constant velocity U in the y-direction. The free surface IJ is a streamline on which $\psi(x, y) = UH$. Here H is the depth of the fluid at $y = -\infty$.

Let the complex potential be $f = \varphi + i\psi$. Without loss of generality we choose $\varphi = 0$ at the corner C. On the free surface IJ where the pressure is constant, the Bernoulli equation yields

$$\frac{1}{2}(\nabla\varphi)^2 - gx = \frac{U^2}{2} + gH. \tag{2.1}$$

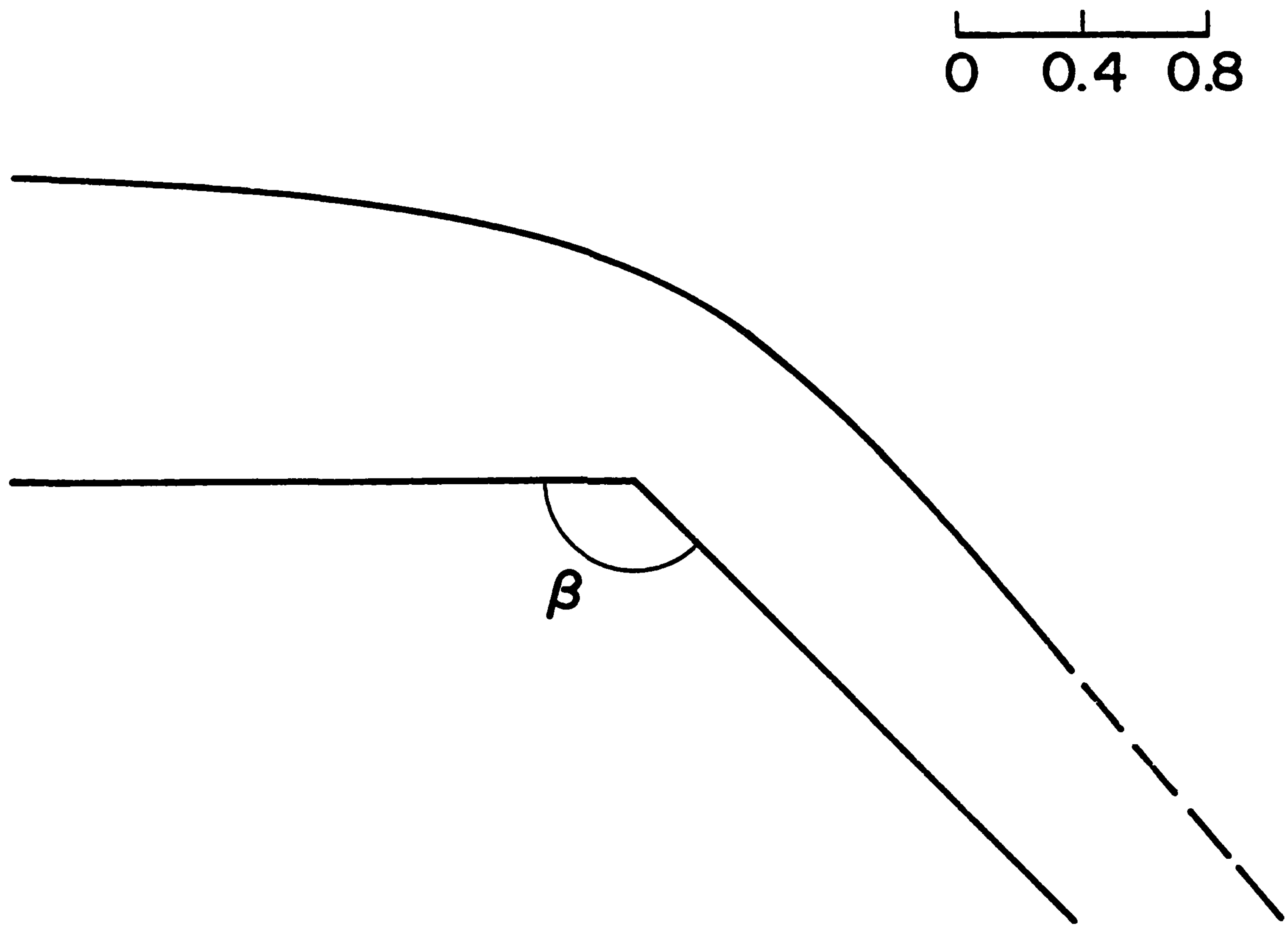

Figure 3. Flow over a spillway with angle β between the two walls. This flow was calculated by the method of Section 2 with $F = 1.3$ and $\beta = 3\pi/4$. The broken line is computed from the asymptotic solution (3.6).

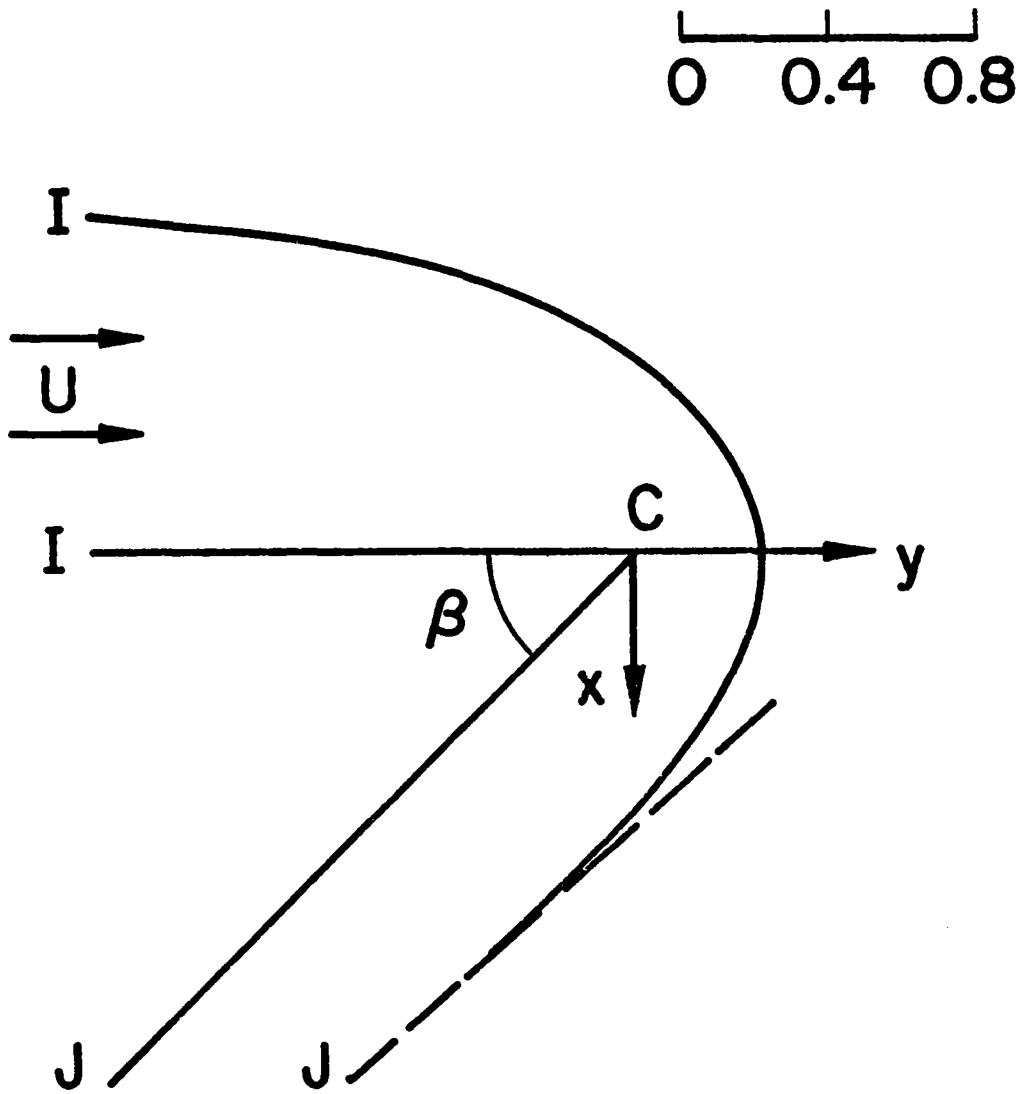

Figure 4. The "teapot effect" flow over a wedge-shaped lip of angle β. The points I and J are at infinity on opposite sides of the wedge. The flow along the horizontal wall has thickness H and velocity U at I. The x,y coordinates have their origin at C, the vertex of the wedge, with x increasing in the downward direction and y increasing horizontally to the right. The streamline shown was computed by the method of Section 2 with $F = 1.3$ and $\beta = \pi/4$. The broken line is computed from the asymptotic solution (3.6).

We choose H as the unit of length and U as the unit of velocity. Then (2.1) becomes

$$(\nabla\varphi)^2 - \frac{2}{F^2}x = 1 + \frac{2}{F^2} \quad \text{on} \quad \psi = 1. \tag{2.2}$$

Here F is the Froude number defined by

$$F = U/(gH)^{1/2}. \tag{2.3}$$

We shall restrict our attention to supercritical flows, for which $F > 1$. The plane of the complex potential $f = \varphi + i\psi$ is shown in Figure 5a.

Let the complex velocity be $\zeta = u - iv$, where u and v are the x and y components of the velocity. For $F^{-1} \neq 0$ (i.e., $g \neq 0$), the velocity ζ increases like $f^{1/3}$ as $\varphi \to \infty$ [4]. Thus we have

$$\zeta \sim f^{1/3} \quad \text{as} \quad \varphi \to +\infty, \qquad F^{-1} \neq 0. \tag{2.4}$$

At $\varphi = -\infty$ the flow is supercritical and is characterized by the presence of exponentially decreasing terms in ζ. Thus ζ has the form

$$\zeta \sim -i[1 + Ae^{\pi\lambda f}] \quad \text{as} \quad \varphi \to -\infty. \tag{2.5}$$

Here A is a constant to be found as part of the solution and λ is the smallest positive root of

$$\pi\lambda - F^{-2}\tan\pi\lambda = 0. \tag{2.6}$$

Near C, where there is a corner of angle β, the flow has the corner behavior

$$\zeta \sim f^{1-\beta/\pi} \quad \text{as} \quad f \to 0. \tag{2.7}$$

The problem is to find ζ as an analytic function of $f = \varphi + i\psi$ in the strip $0 < \psi < 1$, satisfying (2.2), (2.4), (2.5), (2.7) and the kinematic conditions

$$u = 0 \quad \text{on} \quad \psi = 0, \qquad \varphi < 0, \tag{2.8}$$

$$u = -v\tan\beta \quad \text{on} \quad \psi = 0, \qquad \varphi > 0. \tag{2.9}$$

We define the new variable t by the relation

$$f = \frac{2}{\pi}\ln\frac{1+t}{1-t}. \tag{2.10}$$

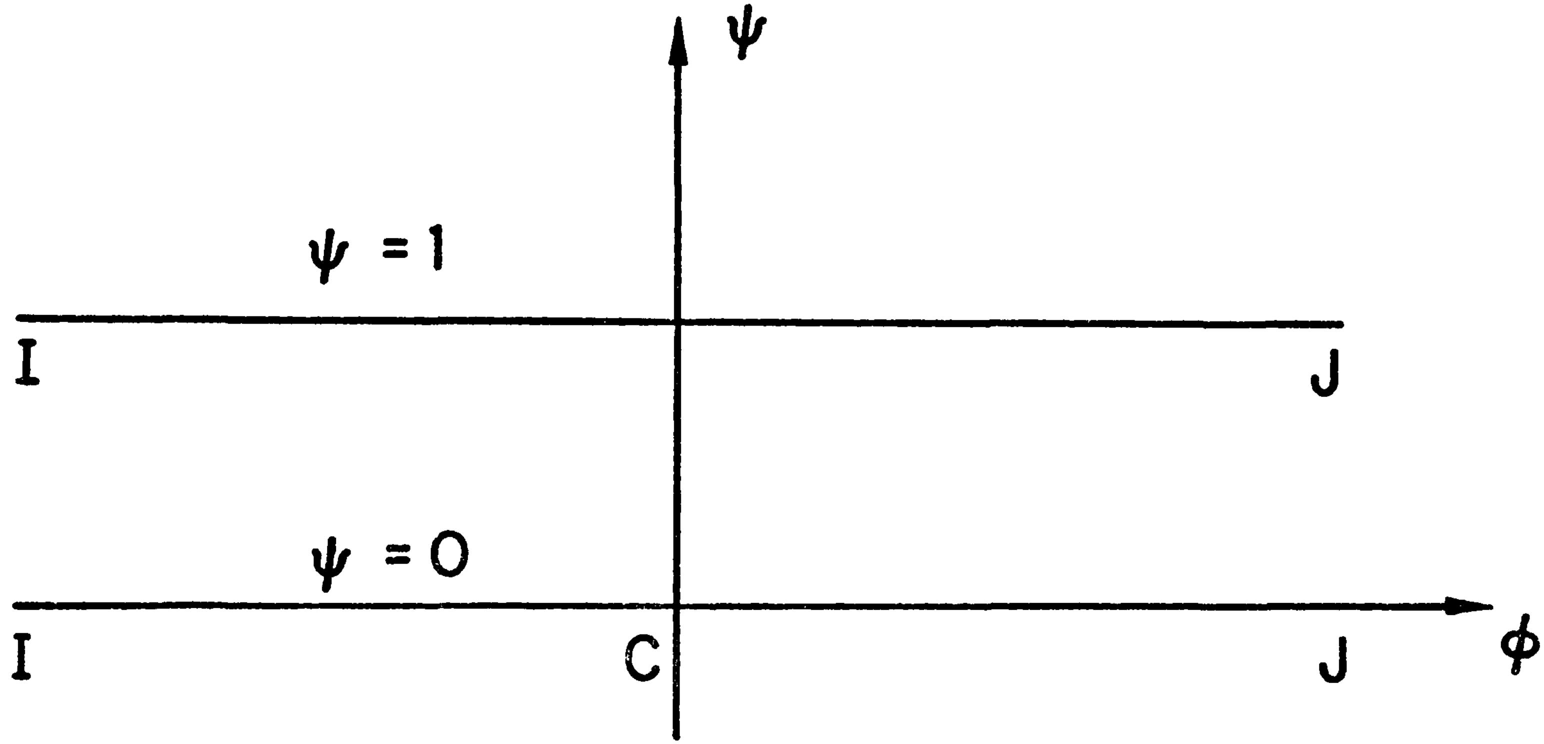

Figure 5a. In the plane of the complex potential $f = \varphi + i\psi$, the flow is confined to the strip $0 \le \psi \le 1$. The images of I, J and C are shown.

The transformation (2.10) maps the flow domain into the interior of the unit circle in the t-plane so that the walls go onto the real diameter and the free surface goes onto that portion of the circumference lying in the upper half of the t plane. (See Figure 5b).

We define the function $\Omega(t)$ by the relation

$$\zeta = -i(-t)^{\beta/\pi-1}[-\ln c(1-t)]^{1/3}[-\ln 2c]^{1/3}[1+(1+t)^{2\lambda}\Omega(t)]. \tag{2.11}$$

Here c is a real constant between 0 and 1/2. We shall choose $c = 0.2$. It can be checked easily that the expression (2.11) satisfies the conditions (2.4), (2.5) and (2.7). The function $\Omega(t)$ is analytic for $|t| < 1$ and continuous for $|t| \leq 1$. The kinematic conditions (2.8) and(2.9) imply that the expansion of $\Omega(t)$ in powers of t has real coefficients.

With this expansion inserted, (2.11) becomes

$$\zeta = -i(-t)^{\beta/\pi-1}[-\ln c(1-t)]^{1/3}[-\ln 2c]^{-1/3}\left[1+(1+t)^{2\lambda}\sum_{n=0}^{\infty} a_n t^n\right] \tag{2.12}$$

For given values of F and β, the unknown coefficients a_n have to be determined to make (2.12) satisfy the Bernoulli condition (2.2).

We use the notation $t = |t|e^{i\sigma}$, so that points on the free surface are given by $t = e^{i\sigma}, 0 < \sigma < \pi$. We find it convenient to eliminate x from (2.2) by differentiating it with respect to σ. This yields $\frac{\partial|\zeta|^2}{\partial\sigma} - \frac{2}{F^2}\frac{\partial x}{\partial\sigma} = 0$. Now on the circular arc $\varphi = \ln(\cot\frac{\sigma}{2})$. Differentiating this relation and using the identity

$$\frac{\partial x}{\partial\varphi} + i\frac{\partial y}{\partial\varphi} = \frac{1}{\zeta} \tag{2.13}$$

we obtain

$$\frac{\partial x}{\partial\sigma} = -\frac{1}{2\sin\sigma}\frac{\tilde{u}(\sigma)}{[\tilde{u}(\sigma)]^2+[\tilde{v}(\sigma)]^2} \tag{2.14}$$

$$\frac{\partial y}{\partial\sigma} = -\frac{1}{2\sin\sigma}\frac{\tilde{v}(\sigma)}{[\tilde{u}(\sigma)]^2+[\tilde{v}(\sigma)]^2} \tag{2.15}$$

Here $\tilde{\zeta}(\sigma) = \tilde{u}(\sigma) - i\tilde{v}(\sigma)$ denotes the value of ζ on the free surface. Upon inserting (2.14) into the differentiated form of the constant pressure condition, we obtain

$$\tilde{u}(\sigma)\tilde{u}_\sigma(\sigma) + \tilde{v}(\sigma)\tilde{v}_\sigma(\sigma) + \frac{F^2}{2\sin\sigma}\frac{\tilde{v}(\sigma)}{[\tilde{u}(\sigma)]^2+[\tilde{v}(\sigma)]^2} = 0 \tag{2.16}$$

We now set $t = e^{i\sigma}$ in (2.12) to get $\tilde{\zeta}(\sigma)$ and substitute that expression into (2.16). We will use the resulting equation to get the unknown coefficients a_n. To do so, we truncate the infinite

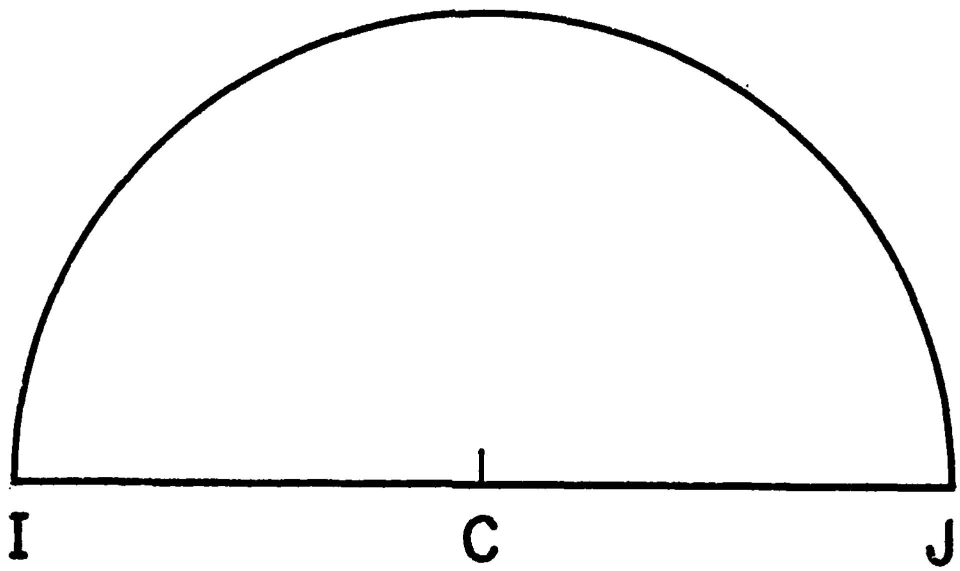

Figure 5b. The image of the strip $0 < \psi < 1$ in the t plane is the semicircle shown here with I, J and C identified.

series in (2.12) after N terms. We find the N coefficients a_n, $n = 0, \dots, N-1$ by collocation. Thus we introduce the N mesh points

$$\sigma_I = \frac{\pi}{2N} + \frac{\pi}{N}(I-1), \qquad I = 1, \dots, N. \tag{2.17}$$

By using (2.12) we obtain $\tilde{\zeta}(\sigma_I)$ and $d\tilde{\zeta}(\sigma_I)/d\sigma$ in terms of the coefficients a_n. Substituting these expressions into (2.16) at the points σ_I, we obtain N nonlinear algebraic equations for the N unknown a_n. This system is solved by Newton's method.

When the coefficients a_n are known for a given F, the values of $\tilde{x}(\sigma)$ and $\tilde{y}(\sigma)$ on the free surface are obtained by integrating (2.14) and (2.15):

$$\tilde{x}(\sigma) = \tilde{x}_0 - \frac{1}{2}\int_{\frac{\pi}{2}}^{\sigma} \tilde{u}(\sigma)[\sin\sigma(\tilde{u}(\sigma)^2 + \tilde{v}(\sigma)^2)]^{-1} d\sigma$$

$$\tilde{y}(\sigma) = \tilde{y}_0 - \frac{1}{2}\int_{\frac{\pi}{2}}^{\sigma} \tilde{v}(\sigma)[\sin\sigma(\tilde{u}(\sigma)^2 + \tilde{v}(\sigma)^2)]^{-1} d\sigma$$

Here $\tilde{x}_0$ and $\tilde{y}_0$ are the values of x and y at $\varphi = 0$, $\psi = 1$. These values are obtained by integrating $\partial x/\partial\psi$ and $\partial y/\partial\psi$ along the equipotential $\varphi = 0$ from $\psi = 0$ (i.e., from the origin) to $\psi = 1$.

3. Free streamline solutions

Before describing our numerical results we shall show that the problem has an explicit solution when $F = \infty$. When $F = \infty$, the Bernoulli condition (2.2) reduces to

$$|\zeta|^2 = 1 \qquad \text{on} \qquad \psi = 1. \tag{3.1}$$

Then (2.4) must be replaced by the requirement that ζ remains bounded as $\varphi \to +\infty$.

It can be checked easily that

$$\zeta = -i(-t)^{\frac{\beta}{\pi}-1} \tag{3.2}$$

satisfies this requirement and also satisfies the conditions (3.1), (2.7)–(2.9). Therefore (3.2) is the solution of the problem for $F = \infty$. Now we eliminate t between (2.10) and (3.2) and use the identity (2.13). Then we obtain after integration

$$z = i\int_0^f \left[-\tanh\frac{(\pi f)}{4}\right]^{1-\beta/\pi} df \tag{3.3}$$

This is the representation of the solution with z given as a function of f. For $\beta = 0$, (3.3) reduces to

$$z = i\int_0^f \left[-\tanh\left(\frac{\pi f}{4}\right)\right] df = -i\frac{4}{\pi}\ln\coth\frac{\pi f}{4}.$$

This is the solution derived by Keller [2]. [See his equation (10).]

A solution for F large can be obtained by considering (3.2) as the first term of an inner expansion of $\zeta(t, F)$ in powers of F^{-2}. This expansion is to be matched to an outer expansion with(2.5) as leading term as $\varphi \to -\infty$ and to another with leading term (2.4) as $\varphi \to +\infty$.

By using (3.2) and (2.10) we find that

$$\zeta \sim -i\left[1 + \left(\frac{\beta}{\pi} - 1\right)e^{\frac{\pi f}{2}}\right] \quad \text{as} \quad \varphi \to -\infty. \tag{3.4}$$

Relation (2.6) shows that $\lambda \to 1/2$ as $F \to \infty$. Therefore matching (2.5) with (3.4) yields

$$A \sim \frac{\beta}{\pi} - 1 \quad \text{as} \quad F \to \infty. \tag{3.5}$$

As $\varphi \to +\infty$, the solution is described by the classical thin jet theory [5]. If δ denotes the thickness of the jet, we have $|\zeta| \sim 1/\delta$. Let us write the equation of the free surface of the jet as $y = y(x)$. Then $\delta = x \cos\beta + y(x)\sin\beta \sim |\zeta|^{-1}$. Substituting these expressions into (2.2) and solving for $y(x)$ yields

$$y = y(x) \sim -x\cot\beta + \frac{F}{\sin\beta}(F^2 + 2 + 2x)^{-1/2}. \tag{3.6}$$

Relation (3.6) is valid for all F provided that $x \gg F^2$. An easy calculation with (2.2), (2.13) and (3.6) shows that the first term in (3.6) (i.e., $y \sim x\cot\beta$) yields (2.4).

The solution (3.2) gives the following expression for the shape of the free surface.

$$y \sim -x\cot\beta + \frac{1}{\sin\beta} \quad \text{as} \quad \varphi \to +\infty \tag{3.7}$$

Therefore (3.7) matches with the leading term in (3.6) for F large.

We have shown that (3.2) matches with (2.4) and (2.5) as $\varphi \to +\infty$ and $\varphi \to -\infty$ respectively. By continuing each expansion to higher order in F^{-2} and matching, we can obtain further terms in the expansions. Together they will represent the flow everywhere for F large.

4. Numerical results

The numerical procedure described in Section 2 was used to compute solutions for various values of F and β. Solutions for $\beta < \pi/2$ illustrate the teapot effect whereas solutions for $\beta > \pi/2$ represent flows in a spillway. Typical profiles with $F = 1.3$ for $\beta = 3\pi/4$ and $\beta = \pi/4$ are shown in Figures 3 and 4 respectively. The broken line corresponds to the asymptotic solution (3.5) for x large. Computed profiles for other values of F and β are qualitatively similar to these two. We were able to find solutions for all the values of $F > 1$ which we tried.

For $F < 1$, (2.6) has purely imaginary roots and then (2.5) corresponds to a train of small amplitude waves at infinity. Although we did not compute them, we expect that there are solutions for $F < 1$ with such wavetrains at infinity.

5. Pouring flows and weir flows

We now consider flows with two free surfaces over a thin wall in water of infinite depth. (See Figure 1.) The flow in Figure 1 with arbitrary wall slope β is a generalization of the flow with $\beta = \pi/2$ calculated by Vanden-Broeck and Keller [1]. Solutions for arbitrary β can be obtained by using the procedure described in Section 3 of their paper with their equation (3.8) replaced by

$$\zeta = -e^{i(\frac{\pi}{2}-\beta)}(1+t)^{2\pi/\beta}[-\ln c(1+t^2)]^{1/3}\exp(\sum_{n=0}^{\infty} U_n t^n) \tag{5.1}$$

For $\beta = \pi/2$, (5.1) reduces to their equation (3.8). A computed profile obtained by using (5.1) with $\beta = \pi/3$ is shown in Figure 1. Similar results obtained with different values of β indicate that there is a unique flow with two free surfaces for each value of β in $0 < \beta \leq \pi/2$. However the analysis presented in the previous sections suggests that there are in addition solutions with one free surface.

In order to compute these extra solutions we consider the flow configuration shown in Figure 2. We choose $\psi = 0$ on the wall ICJ and $\varphi = 0$ at the point C. Let Q be the value of ψ on the free surface. We introduce dimensionless variables such that $Q = g = 1$. In these variables, the Bernoulli equation yields

$$(\nabla\varphi)^2 - 2x = 0 \quad \text{on} \quad \psi = 1. \tag{5.2}$$

The complex potential plane is shown in Figure 5a. By using the transformation (2.10) as before, we map the flow domain into the interior of the unit circle in the t-plane shown in Figure 5b. It can be shown that the following expression is a suitable expansion for the complex velocity:

$$\zeta \sim e^{i(\frac{\pi}{2}-\beta)}(1+t)^{\frac{2\beta}{\pi}}t^{-1}[-\ln c(1+t)]^{1/3}\exp\left(\sum_{n=0}^{\infty} U_n t^n\right) \tag{5.3}$$

Here c is an arbitrary constant between 0 and 0.5. As before we choose $c = 0.2$. The coefficients U_n are determined by the collocation procedure of Section 2 with the parameter F in (2.16) replaced by 1. A typical profile for $\beta = \pi/3$ is shown in Figure 2.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Free surface flows of a liquid poured from a container are calculated numerically for various configurations of the lip. The flow is assumed to be steady, two dimensional and irrotational, the liquid is treated as inviscid and incompressible, and gravity is taken into account. It is shown that there are flows which follow along the under side of the lip or spout, as in the well-known "teapot effect", which was treated previously without including gravity. Some of the results are applicable also to flows over weirs and spillways.

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